SUBORDINATE DEMONSTRATIVE SCIENCE IN THE SIXTH BOOK OF ARISTOTLE'S *PHYSICS*

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FEW interpreters of Aristotle have denied that both empirical, inductive methods and some kind of systematic deduction played a role in the philosophy of the biologist who expounded the West's first formal logic. But it has usually been the fashion to focus on one side of this polarity. In recent decades the focus has been on the empirical Aristotle. But some of the latest studies emphasize that Aristotle varied his methods according to context. G. E. L. Owen,[1] for example, although he feels that Aristotle took little care to separate inductive and deductive methods in the treatises, does allow that he uses deduction at times, and mentions book 6 of the *Physics* among them.[2]

The general trend against interpreting Aristotle's work as the deductive pronouncements of an armchair philosopher has been particularly fruitful as regards the *Physics*. But it is not insignificant that scholarship in this trend has treated primarily *Physics* 1–4.[3] The approach which Aristotle uses in the rest of the *Physics* is much more deductive. Book 6, in particular, contrasts strikingly with the methods which prevail in the first four books.

There is no searching for definitions in book 6. Nor is there any exploratory analysis of linguistic phenomena or of current philosophical theories. Language serves only a few times as a casual source for definitions from which deductions may be made ($234^{a}32$, $234^{b}5$, $239^{a}15$, and $239^{a}27$), and other philosophical theories, with the exception of Zeno's paradoxes, are alluded to but once ($233^{a}13$). The book is almost entirely composed of a series of proofs, deduced from previous definitions and conclusions. The proofs are not reduced to syllogisms, but the reasoning is strictly deductive. There is generally even a format which reminds the reader of Euclidean geometry; each theorem to be proved is first stated (usually after a few lines of preliminary assumptions or reasoning), then proved, and finally explicitly repeated as a conclusion. The only interruptions in this chain of proofs are some brief passages of explanation ($233^{a}13$–21, $235^{a}34$–5, $236^{a}7$–14, $236^{b}1$–18, $237^{a}35$–$^{b}22$, $239^{a}20$–22, $240^{b}8$–20, and $241^{b}11$–20) and the discussions of Zeno ($233^{a}21$–33, and all of ch. 9).

The vocabulary of the book reflects its contents. There is not one mention of ἀπορία. Instead, the theories are regularly introduced as δῆλον or φανερόν and repeated as δεδειγμένον. The necessity and universality of the arguments are kept obvious by the constant use of modal qualifiers such as ἀνάγκη and ἀδύνατον and universal quantifiers such as πᾶς and ἅπας. Finally, Aristotle repeatedly refers to what he is doing as ἀπόδειξις ($233^{a}7$, $233^{b}14$, $237^{a}35$, $238^{a}32$, $238^{b}16$, $238^{b}26$, and $240^{b}8$).[4]

[1] G. E. L. Owen, τιθέναι τὰ φαινόμενα, in *Aristote et les problèmes de méthode* . . . (first) *Symposium Aristotelicum* (Louvain, 1961), 83–103.

[2] Owen, op. cit. 84.

[3] e.g., W. Wieland, *Die Aristotelische Physik* (Göttingen, 1962).

[4] The last instance does follow the close of

All this inevitably reminds one of the theory of ἐπιστήμη ἀποδεικτική ('demonstrative knowledge' or 'science'), an axiomatic system of deductive theorems about a certain class of entities, described in the first book of the *Posterior Analytics*. But it is widely believed that no such deductive system can be found in Aristotle's own treatises. Consequently, the theory of demonstration is generally either ignored or studied in isolation from its author's own methodological practice.

One reason for the belief that Aristotle never used the method is the expectation that a demonstrative system should employ only exactly formulated syllogistic arguments, corresponding transparently to the modes and figures of the *Prior Analytics*. In the *Posterior Analytics*, Aristotle seems to define a demonstration as a necessary syllogism, then proceed to articulate several structural requirements for a systematic science which will render its syllogisms necessary. In the treatises, however, few of Aristotle's arguments are formulated as syllogisms.[1] So, plausibly assuming that where there are no syllogisms, there can be no axiomatic system composed of syllogisms, one refuses to embark on any search for demonstrative science in the treatises. Instead, one feels compelled to seek some explanation for the paradoxical construction of a methodological theory which its creator never tried to apply.

But is the absence of exact syllogisms in the treatises really sufficient reason for concluding that the entire first book of the *Posterior Analytics* is irrelevant to the rest of Aristotle's work? The *Analytics*, after all, specifies several other requirements for demonstrative science. If an extensive text of deductive argument in the treatises is found to fulfil all these other requirements, it seems unreasonable to deny that it was written according to the method. However the lack of syllogistic formulation is explained, it could not warrant the denial that the method is at least roughly applied in such a text.

And in fact, when one considers the literary form of the treatises, it seems unlikely that much more than such a rough application could be expected. These are lecture notes, and even if Aristotle did elaborate a complete series of syllogisms for a science, one might still expect to find in these informal lecture summaries no more than a sketchy presentation of the system, resembling it faithfully only in its general structure. Philologically, it is hardly a sound procedure to discount the possibility.

The burden of this paper is to submit that *Physics* 6 is just such a text. No further explanation of its lack of syllogistic formulation besides the literary genre of the treatises will be attempted here. Such an attempt would involve a thorough interpretation of the *Analytics*, which would not only exceed the scope of this article, but run counter to its purpose. That purpose is to show that there is an unmistakable correspondence between the book's contents and the *Analytics*' theory, which must be taken into account in interpreting either work.

In the *Posterior Analytics*, Aristotle tried to structure his theory of demonstrative science so as to fulfil the familiar Platonic criteria for ἐπιστήμη: univer-

ch. 9, which is the longest non-demonstrative passage in the book, but the plural ἀποδεδειγμένων τούτων, as it marks the close of the discussion of Zeno, may be taken to refer not to the immediately preceding section, which draws a distinction rather than giving any demonstration, but to the entire correction of Zeno's errors implied throughout the first nine chapters of the book.

[1] e.g., W. Wieland, op. cit. 42, n. 1. Cf. G. Patzig, *Die Aristotelische Syllogistik* (Göttingen, 1959), 201.

sality, necessity, and the ability to give an account of what is known, i.e., knowing why it is so.

His demonstrative science assumes the definition and existence of a restricted genus which it takes as its subject, and it also assumes the definitions of the properties of its subject. Beyond that, the only permissible assumptions are the principles common to all or several sciences, such as the principle of contradiction. He allows, however, that a science may neglect to define or assume explicitly the existence of its subject, to define the properties, or to state the meanings of common principles, whenever these are obvious. The properties should be definitionally related to the subject 'καθ' αὑτό', i.e., either they must be implied in its definition or it must be implied in their definitions. The science proves the existence of the properties, or their predicability of the subject, and the interrelations of the properties. It deduces its theorems by demonstrations from the definitions assumed and from theorems previously deduced from them, with the help of the common principles (*Posterior Analytics* I $73^{a}34$–$^{b}5$, $76^{a}31$–$^{b}1$).

The definitional nature of the premisses and the logical correctness of the deductions guarantee the universality and necessity of the theorems within the class under study. And since the premisses and the conclusion are both about definitionally related properties of the same genus, the knowledge which results will include an understanding of the formal cause of the conclusion.

I have mentioned that *Physics* 6 consists almost entirely of a series of proofs deduced from definitions. We may, then, begin the comparison of *Physics* 6 with the standards of the *Posterior Analytics* by asking whether the terms from whose definitions these proofs are deduced are definitionally related properties of a specific genus. This will be done in section (1) which follows. Section (2) will examine the contents of *Physics* 6, to ascertain whether they are concerned with the existence and interrelations of the properties of that genus. And section (3) will compare the methods of demonstration in the book with those expected in the *Posterior Analytics*. The results of these three sections, though generally positive, will reveal certain discrepancies between the *Physics*' practice and the *Analytics*' standards. But the remainder of the paper will show that these discrepancies are explained by the fact that what we have here is not an ordinary science, but what Aristotle calls a subordinate one.

1. Most of the defined terms operative in the book's reasoning fall into two definitional groups, one beginning with *change*, and the other ending with *continuous*. These are as follows:

I	II
change or *motion* (book 6, unlike 5, usually uses κίνησις and μεταβολή indiscriminately)	*between*
becoming, *locomotion*, etc. (species of change)	*successive*
rest	*contiguous*
time	*together*
moment	*touching*
faster	*continuous*

Let us examine the first of these groups. Most of these terms are defined either in book 6 or in book 5, which was composed with book 6 and prepares for it. *Motion* is, of course, defined in book 2 (201^{a}11), and its definition is explicitly presupposed in book 5 (224^{b}10). The species of change are defined at the start of book 5 also. *Moment* is defined in book 4 (222^{a}33), and book 6 repeats the definition in 233^{b}35 and makes deductions from it. The definition of *rest* used in the first half of the *Physics* is given in book 5 too (226^{b}14), and it is repeated for use in the proofs of book 6 (234^{a}32, 239^{a}13), though book 6 also introduces an alternative definition of it (234^{b}5, 239^{a}15).

All the terms in the family are καθ' αὑτό to *change*. *Change* (or *motion*) is included in the definitions of its species, of *rest*, of *time* and of *faster*, and *moment* is defined in terms of *time*.

The only difficulties for the expectations of the *Posterior Analytics* in regard to this group are that the definition of time laboriously worked out in book 4 is neither referred to nor used as a principle of deduction in books 5–6, and that of motion worked out in book 2, though presupposed in book 5 (224^{b}10), is not repeated or used as a principle of deduction. Of course, Aristotle might not have discovered those definitions when he first wrote books 5 and 6; but there is more to it than that.

Definitions of time and change can be said to be used by book 6, but only in a peculiarly limited way. The book states certain assumptions as preliminaries to its proofs. The most important of these are two basic statements about time and change which are used repeatedly and seem to function in the way we should expect their definitions to function. They may even be remnants of definitions which were given in the original version of books 5–6. The assumption that 'all change is from something into something' (234^{b}10, 235^{b}6, 239^{a}23) is derived in book 5 (225^{a}1–2) from a verbal definition of μεταβολή, which is in fact the only definition of change which book 5 explicitly gives. And the assumption that 'all change is in time' (232^{b}20, 236^{b}19, 237^{b}23, 239^{a}23), which is also treated as basic in book 5 (224^{a}35, 227^{b}26), is made definitional in book 4, 221^{a}4–7, which explains that for motion, what it is to be 'in time' is to be measured by it. For Aristotle defines time as the measure of motion.[1]

Let us turn now to the other group of terms. These are all defined in book 5, ch. 3. The definitions given there are used as principles of deduction at the very start of book 6, and 5. 3 has no obvious relevance to its own context, so one may safely conclude that it was written for the sake of book 6. Interestingly, the corresponding chapter in *Metaphysics K* (ch. 12) does stand at the end of that book. The significance of 5. 3 for 6 has seldom gone unnoticed. Particularly interesting for our purposes, however, is the comment of Aquinas:

> Praemittit autem haec, quia horum definitionibus utitur in demonstrationibus consequentibus per totum librum; sicut et in principio Euclidis ponuntur definitiones, quae sunt sequentium demonstrationum principia.
>
> (Commentary on the *Physics*, *ad loc.*)

It is clear that the terms of this group are definitionally interrelated, for that is how 5. 3 defines them.

[1] Admittedly, there is also an attempt to prove this assumption, in *Physics* 222^{b}30–223^{a}15.

What is not clear at first glance, however, is how the second group of terms can join the first in one genus. The terms in the first group, being all καθ' αὑτό to *change*, are clearly appropriate to Aristotelian physics. But the terms in the second group seem to be at least as appropriate to geometry as to physics, and they are not really καθ' αὑτό to *change*. It is true that Aristotle does forge a definitional link between the two groups by the way in which he handles *between* in 5. 3. *Between* appears in the definition of *successive*, and therefore indirectly in those of *contiguous* and *continuous*; but *between* itself is defined in terms of motion. Yet this does not reduce the properties from *successive* to *continuous* to full membership in the physical genus. For *between* functions in the definition of *successive* only negatively. *Successive* is 'what has nothing of its own kind between itself and that to which it is successive' (227^a1, an unfortunately circular definition). Hence it is *successive*, and not *between*, that is the first principle of the genus of properties ending in *continuous* (227^a17).

Thus the two families of defined terms involved in the book represent two different genera.[1]

Several undefined terms are also operative in the book's proofs, and these reflect the same duality. The undefined mathematical terms are *whole*, *part*, *equal*, *unequal*, *greater*, *less*, *divisible*, *indivisible*, *limit* (πέρας), *extremity* (ἔσχατον), *finite*, *infinite*, *extension* (μέγεθος), etc. These all have obvious meanings, so they need not be defined. The undefined physical terms have equally obvious meanings, which become clear in the light of related terms which are defined. *Standing*, for example, is equated with 'coming to rest' (ἠρεμίζεσθαι, 238^b25), and *rest* is defined. Similarly, *slower* and *of the same speed* are correlative with *faster*, and *uniform* appears in the definition of *faster*. The definition of *faster* is given fully in 222^b33 and recalled as a principle of deduction in 232^a26.[2]

The terms in the book and their definitions, then, reveal a pattern of deduction like that of a demonstrative science, but with the following apparent discrepancies: the book uses mathematical concepts as well as those proper to physics; the definitions of *change* and *time*, the most basic physical concepts of the book, can be said to be used only in an indirect way; and the properties which the book studies fall into two distinct, though related, families.

Of course, we must also ask whether book 6 makes deductions from any proposition other than the definitions and assumptions warranted by the *Posterior Analytics*.

The assumptions which the book makes as preliminaries to certain proofs seem to pose some threat to its demonstrative validity in this regard, but hardly a serious one. I have shown how the two most important assumptions are derived from definitions of change and time. The rest, too, can usually, if not always, be derived from warranted principles. The statement that 'what is between moments is time' (231^b9, 237^a5), for example, could be deduced from the definition of *moment* in terms of time and from the statement in 234^a8 that every continuum is such that there is something of the same nature between its limits, which in turn could be derived from the definition of *continuous*. And the statement in 237^a19 that 'all that has changed from something into something has changed in time' simply combines the basic assumptions about change and time.

[1] The mathematical character of the book has been noticed, e.g. by J. M. Leblond, *Logique et méthode chez Aristote* (Paris, 1939), 197.

[2] Both *faster* and *slower* are defined in 215^b15, but book 6 defines only *faster*.

Such assumptions do constitute a minor flaw, given the *Analytics*' stipulation that every middle term in a demonstrative science should be explicit. But in his lecture summaries Aristotle neglects the obvious.

2. In the *Posterior Analytics*, Aristotle expects a science to demonstrate whether the properties exist, or whether they inhere in the subject, and the interrelations of the properties. In a way, *Physics* 6 certainly does so. Moments are proved to inhere in time but to preclude change (233^b33–234^a31). *Faster* and *slower* are related to change and time (232^a25–232^b20). And while in certain senses what can be called 'the first time' exists, in others it does not (236^a7–14). Yet while the *Posterior Analytics* implies an exhaustive picture of the relations within one class, *Physics* 6, as we have seen, involves two classes. For while *change*, *time*, and their properties are physical concepts, *continuity* and *extension* are normally mathematical concepts. In fact for Aristotle, extension, defined as continuous quantity, is the subject of geometry (*Metaphysics* 1020^a7–12). But *Physics* 6 is an exhaustive study of the interrelations of time, change, and extension *qua* continua. Its first principle is not the definition of change, which one would expect in the *Physics*, but the attribution to change of the property of infinite divisibility. This explains the limited way in which definitions of time and change are brought into the book through the basic assumptions that change is 'from something into something' (ἔκ τινος εἴς τι) and 'in time' (i.e., measured by it). These statements tell little about the essence of change and time. What they do tell is that change is commensurate with extension and time is commensurate with change.

In the book, the interrelations of time, change, and extension entail an Aristotelian view of physical reality and preclude the difficulties created by the famous paradoxes of Zeno.

Aristotle first proves that continua are by nature infinitely divisible, then that extension, change, and time are commensurate and must all together be either continuous or composed of indivisible units. Hence the division of any one of these three entails that of the others (ch. 1). It is this mutual divisibility that concerns him most in the opening chapters, and he uses it throughout the book to prove his theorems on the continuity of the three commensurates by having the division of one entail that of another.

After he has proved the mutual entailment of division among the three commensurates, he introduces theorems about the terms *faster* and *slower* which will help him to show how the commensurates divide each other (ch. 2). He then proves that time, and consequently extension, are continuous. In ch. 3, he introduces the moment, the indivisible in time, and theorems excluding the possibility of change occurring within it. And in ch. 4, he proves that change and its attributes are continuous.

In the rest of the book, he uses the mutual continuity now established to prove theses rounding out a picture of change that not only precludes Zeno's, but provides an excellent matrix for his own theories about nature, especially about generation, corruption, and change of quality. This purpose is revealed when he applies what he is proving particularly to these kinds of change (236^b1–18, 237^b9–20, 238^a19).

Chapters 5–6, for example, use the concept of 'first (πρῶτος) time' to establish several relations between time and change. Thus, in the theorem 'there is no first time in which a thing changed' (236^a14), the beginning of change is viewed as continuous, to evade Zeno's menace. But in the theorem 'the first time in which what has changed has changed must be indivisible' (235^b32), the termination of change is viewed as a discrete moment. This provides for Aristotle's own forms, which come into being instantaneously.

Ch. 7 extends the demand that all three commensurates (time, change, and extension) be considered 'finite' (i.e., as discrete quantities) or 'infinite' (i.e., as infinitely divisible continua) together. Its theorems show that combinations of an infinite commensurate with a finite one are impossible; for when the division of the finite one by its finite parts exhausts it, the corresponding finite parts of the other one must exhaust it too. Ch. 8 applies several theorems proved about change to standing, and ch. 10, after a final disjunction of change and indivisibility, concludes with a discussion which prepares for the theological problems of books 7–8.

Physics 6, then, really concerns the interrelations of two classes of properties. However, we shall see later that this does not in fact deviate from the norms of the *Posterior Analytics*.

But what about the treatment of Zeno? The criticism of the Eleatic might seem to be a dialectical, rather than a demonstrative, approach. The *Analytics* does, however, provide for one kind of polemical interest:

> τὸν δὲ μέλλοντα ἕξειν τὴν ἐπιστήμην τὴν δι' ἀποδείξεως οὐ μόνον δεῖ τὰς ἀρχὰς μᾶλλον γνωρίζειν καὶ μᾶλλον αὐταῖς πιστεύειν ἢ τῷ δεικνυμένῳ, ἀλλὰ μηδ' ἄλλο αὐτῷ πιστότερον εἶναι μηδὲ γνωριμώτερον τῶν ἀντικειμένων ταῖς ἀρχαῖς ἐξ ὧν ἔσται συλλογισμὸς ὁ τῆς ἐναντίας ἀπάτης, εἴπερ δεῖ τὸν ἐπιστάμενον ἁπλῶς ἀμετάπειστον εἶναι. (72^a37–$^b3$)

Let us review the treatment of Zeno in *Physics* 6 in the light of this passage.

In ch. 2, Zeno is introduced abruptly (233^a21); his relevance is obvious. The way in which Aristotle deals with him shows why. After stating that Zeno's false assumption was that it would be impossible to traverse an 'infinite' (i.e., continuous) extension in a finite time, he explains that both commensurates will be infinite in divisibility, but finite in length. He himself then goes on to prove that, in the proper sense, it is indeed impossible, not only to traverse the infinite in a finite time, but even to traverse the finite in an infinite time. Thus Zeno's mistake was precisely to start from an assumption contradictory to Aristotle's basic thesis of mutual divisibility, which he has just proved in ch. 1.

Ch. 9 continues the discussion of Zeno in the same fashion. His paradoxes are sketchily summarized, but not systematically refuted. Instead, Aristotle simply indicates how they assume principles contradicting his own. Zeno's errors are either to assume that one commensurate may be considered 'finite' while the other is considered 'infinite' (as in ch. 2), or to assume that time is composed of indivisible moments and conclude that motion is composed of indivisible jerks (which contradicts Aristotle's basic attribution of the property of continuity to both).[1]

3. One of the Platonic criteria of knowledge which the *Posterior Analytics* tries to fulfil is knowing why a thing is so. Aristotle therefore isolates as the best

[1] Similarly, in 240^a19 ff., when the traditional problem of quality change (that a changing subject both has and does not have the quality in process) is solved by a distinction between the whole and the parts of the thing, Aristotle is depending on his basic principle that anything which changes is always divisible into parts.

kind of demonstration the kind that provides knowledge '*τοῦ διότι*'. He expects that such demonstrations will generally consist of direct deductions through first-figure syllogisms. Some scholars have concluded that a demonstrative science must be composed solely of first-figure syllogisms. But Aristotle clearly treats such demonstrations only as 'better' (cf. 85^{a}13–15), not as alone valid. When he discusses the details of demonstrative science, such as the limited number of middle terms, he discusses all three syllogistic figures (82^{b}29–33).

Similarly, he prefers direct deductions to indirect proofs such as reductions, and he prefers proofs using only positive premisses to those which also use negative ones. But despite these preferences, he examines other patterns of argument which he considers inferior, but legitimate. These, though neglected by recent scholarship, are most relevant for understanding the demonstrative method as it is applied in *Physics* 6.

Hence there are two questions which we may ask about the proofs in *Physics* 6:

1. Are they syllogistic (especially in BARBARA, the universal, positive mode of the first figure)?
2. To what extent do they follow patterns of demonstration allowed by the *Posterior Analytics*?

The first question has already been answered. The book is no series of clearly formulated, explicit syllogisms. There are, to be sure, some passages where the reasoning follows an easily discernible syllogistic pattern. In 232^{b}20–23, for example, we find the following:

> *ἐπεὶ δὲ πᾶσα μὲν κίνησις ἐν χρόνῳ καὶ ἐν ἅπαντι χρόνῳ δυνατὸν κινηθῆναι, πᾶν δὲ τὸ κινούμενον ἐνδέχεται καὶ θᾶττον κινεῖσθαι καὶ βραδύτερον, ἐν ἅπαντι χρόνῳ ἔσται τὸ θᾶττον κινεῖσθαι καὶ βραδύτερον.*

Here '*ἔσται*' is a mere stylistic variation for '*ἐνδέχεται*', and '*πᾶσα . . . κίνησις ἐν χρόνῳ*', though relevant information, is not operative in the reasoning; otherwise, the sequence in BARBARA is clear. And 238^{b}27–30 offers two more deductions in BARBARA with even less adjustment:

> *τὸ γὰρ κινούμενον ἐν χρόνῳ κινεῖται, τὸ δ' ἱστάμενον δέδεικται κινούμενον, ὥστε ἀνάγκη ἐν χρόνῳ ἵστασθαι· ἔτι δ' εἰ τὸ μὲν θᾶττον καὶ βραδύτερον ἐν χρόνῳ λέγομεν, ἵστασθαι δ' ἔστιν θᾶττον καὶ βραδύτερον.*

Usually, however, the proofs are not so obviously syllogistic as in these passages.

But our consideration of the second question will yield more positive results.

If direct demonstrations are rare in the book, the contrary is true of reductions. In the *Posterior Analytics*, Aristotle finds this indirect kind of demonstration less desirable, but permissible, and even—when what is most evident is the falsity of what is entailed by the contradiction of the demonstrandum—most suitable (*Posterior Analytics* 1. 26).

Generally the *Analytics* bans demonstrations which deduce from the principle of contradiction, which is common to all sciences, as violating the restricted limits of a science and hence being more appropriate to dialectic. Aristotle does allow an exception for reductions, but even they, he says, will usually assume only the specific application of the principle of contradiction which touches their subject (77^{a}10–12 and 22–25). This would be the case, for example, when the alternatives are definitionally καθ' αὑτό to the subject, as *odd* and *even* to *number*. One might, then, consider it a confirmation that *Physics* 6 is following the methodology of the *Analytics* if the alternatives in its reductions are such.

When one investigates to what extent they are, of course, one finds the same combination of properly physical material with mathematical material which was revealed by the survey of the terms used in the book. In 237^{a}12, for example, it is argued that if an object which passes through an uninterrupted motion must at a given time either be changing or have changed, then at an indivisible moment, since it cannot be changing, it must have changed. The alternatives 'be changing' and 'have changed' are καθ' αὑτό to *change* and therefore appropriate to physics. But there are other reductions whose alternatives are less physical than mathematical. For example, in 232^{b}14–20 what is faster is shown to move in less time, rather than in equal or greater time. *Greater*, *equal*, and *less* are mathematical alternatives.[1]

There is another way in which *Physics* 6 uses proof patterns which are legitimate but inferior according to the *Posterior Analytics*. I have mentioned Aristotle's preference for demonstrations 'τοῦ διότι'. These he contrasts with demonstrations 'τοῦ ὅτι'. He gives the following examples:

> Planets do not twinkle; what does not twinkle is near; therefore planets are near.
>
> Planets are near; what is near does not twinkle; therefore planets do not twinkle.

The first is τοῦ ὅτι, the second τοῦ διότι. The planets are not near *because* they do not twinkle; they fail to twinkle because they are near. In a demonstration τοῦ διότι, the premisses state the cause of the conclusion.

Though Aristotle views demonstrations τοῦ διότι as better, he regards demonstrations τοῦ ὅτι as legitimate proofs. Some scholars seem to think that he envisages the use of the latter only by error or only in non-demonstrative disciplines which he calls 'ἐπιστῆμαι' only in a loose sense of the word. But Aristotle specifies occasions for their legitimate use (78^{a}28–30, 78^{b}11–13), and he foresees their occurrence 'ἐν τῇ αὐτῇ ἐπιστήμῃ' which uses proofs τοῦ διότι (78^{a}23).[2]

Now, the extent to which *Physics* 6 uses proofs τοῦ διότι and proofs τοῦ ὅτι can be determined by comparing the book with the following passages from *Physics* 3:

> τὸ δ' ἄπειρον οὐ ταὐτὸν ἐν μεγέθει καὶ κινήσει καὶ χρόνῳ, ὡς μία τις φύσις,

[1] There is one reduction whose alternatives are those of the universal principle of contradiction, but in this peculiar case the universal principle is itself proper to the subject, the argument being about the change of opposites (235^{b}13–17).

[2] One might well ask how demonstrations τοῦ ὅτι can be admitted in a system in which a criterion of knowledge is the ability to give the reasons for the facts. Probably it is because Aristotle expects that any premisses which are necessary and which link terms defined καθ' αὑτό to the subject must give at least some insight into the formal cause of the conclusion. Of course, that argument would not be entirely valid in subordinate sciences, which study mathematical properties inherent in physical subjects to which they are not καθ' αὑτό (cf. p. 289). But even subordinate sciences, in so far as they study mathematical properties, grasp some aspect of formal causality. Hence Aristotle can characterize the same science, optics, as τοῦ ὅτι in relation to mathematics, but τοῦ διότι in relation to 'τὸ περὶ τῆς ἴριδος' in 79^{a}10–13.

ἀλλὰ τὸ ὕστερον λέγεται κατὰ τὸ πρότερον, οἷον κίνησις μὲν ὅτι τὸ μέγεθος ἐφ' οὗ κινεῖται ἢ ἀλλοιοῦται ἢ αὐξάνεται, ὁ χρόνος δὲ διὰ τὴν κίνησιν. νῦν μὲν οὖν χρώμεθα τούτοις, ὕστερον δὲ ἐροῦμεν καὶ τί ἐστιν ἕκαστον, καὶ διότι πᾶν μέγεθος εἰς μεγέθη διαιρετόν. (207ᵇ21–27)

This passage predicts a demonstration τοῦ διότι for the infinite divisibility of extension, and establishes an order of priority among the commensurates which would have to be observed in demonstrations τοῦ διότι of their infinite divisibility. In book 6, Aristotle does give a direct demonstration of extension's infinite divisibility (232ᵃ23–25). It, combined with the first two theorems of ch. 1, which deduce the infinite divisibility of continua from the definitions in 5. 3, may be considered a demonstration τοῦ διότι. And in the opening demonstration of the mutual entailment of divisibility among the three commensurates, motion's divisibility is derived from that of extension, and that of time is third, which is the order of priority given in our passage. In the remainder of book 6, however, Aristotle relies on this mutual entailment to divide any commensurate by any other. In 233ᵃ10–12, for example, extension's division is derived from that of time, and in 235ᵃ10–13, motion is proved divisible by time. Aristotle's technique of relying on mutual divisibility deflects his attention from the proofs τοῦ διότι of his thesis, which would have had to concentrate (as ch. 1 alone does) on the nature of continuity itself, and results in an emphasis on demonstrations τοῦ ὅτι showing the relations of these three continua and the consequences of their continuity. For it is such facts, more than their ultimate explanation in the nature of continuity, that are most relevant to his purpose. He is less concerned with the essential nature of either continuity or motion than with the consequences of their liaison.

That is why he gives in book 6 a reply to Zeno which he corrects in book 8 as not getting to the roots of the matter. The answer in 8 is couched in terms of potency and act, but it also differs from that of 6 because it abandons the procedure of entailing division from one commensurate to another and explains the nature of continuity in one commensurate alone:

αὕτη ἡ λύσις (*sc.* that in book 6) πρὸς . . . τὸ πρᾶγμα καὶ τὴν ἀλήθειαν οὐχ ἱκανῶς· ἂν γάρ τις ἀφέμενος τοῦ μήκους καὶ τοῦ ἐρωτᾶν εἰ ἐν πεπερασμένῳ χρόνῳ ἐνδέχεται ἄπειρα διεξελθεῖν, πυνθάνηται ἐπ' αὐτοῦ τοῦ χρόνου ταῦτα (ἔχει γὰρ ὁ χρόνος ἀπείρους διαιρέσεις), οὐκέτι ἱκανὴ ἔσται αὕτη ἡ λύσις, ἀλλὰ τὸ ἀληθὲς λεκτέον . . . (263ᵃ15–22)

Thus the argument is merely τοῦ ὅτι in 6, while 8 reveals τὸ διότι.

In fact, so many of the proofs in 6 are τοῦ ὅτι that Aristotle must interrupt the systematic deductions several times to explain the reason behind them (235ᵃ34–ᵇ6, 236ᵇ1–18, 237ᵃ35–ᵇ22, 239ᵃ20–22).

To summarize, the study of *Physics* 6 has revealed that it does follow the structure of demonstrative science given in the *Posterior Analytics*, but with certain exceptional characteristics. Most significantly, the book studies mathematical concepts as well as physical ones. It concerns two classes of entities, and it studies exhaustively neither class, but their interrelations. And because of this dual subject, most of the book's demonstrations are demonstrations τοῦ ὅτι.

But all these characteristics merely indicate that the method applied here is that of the quasi-mathematical sciences which Aristotle provides for as an exception to the rule restricting a demonstrative science to a single genus; i.e., subordinate sciences.

The usual examples of subordinate sciences in the *Analytics* are optics, harmonics, etc. Their subjects are intermediate between two genera, and their demonstrations are predominantly τοῦ ὅτι. An ordinary science manifests the formal cause of its conclusions by deducing them from the definitions of the genus and properties which it studies. But these subordinate sciences treat facts whose causes must be explained by another, purely mathematical science, because they study properties of a mathematical class inherent in non-mathematical subjects ($76^{a}11$–13, $78^{b}33$–$79^{a}16$).

Since, as we have noticed, *continuity* and *extension* are mathematical concepts while *motion* and *time* are physical ones, the study of motion and time as continua commensurate with extension must be just such a subordinate science. There are two objections to this suggestion:

1. Why does Aristotle only give optics, astronomy, etc. as examples of such sciences and never mention his own study in *Physics* 6?

2. Would not the appearance of a subordinate science in the *Physics* violate Aristotle's strict separation of physics and mathematics and imply a world-view which, like that for which he sometimes criticizes the Pythagoreans, reduces the essence of physical things to mathematics?

In a way, the answer to both objections is the same. What I am suggesting is not that *Physics* 6 constitutes a separate science independent of the rest of the *Physics*, but that in it Aristotle offers a systematic, deductive study, using the methodology of subordinate sciences, of one aspect of the analysis of change; because, in that particular aspect, change is related to a mathematical property, and he can best present that relation and its consequences through such a study. Harmonics and optics were independent disciplines to which Aristotle could point as obvious areas for subordinate sciences. What he does in *Physics* 6 is perhaps more comparable to the liaison between mathematics and medicine described in $79^{a}13$–16. These two disciplines are linked not by any complete subordinate science, but only in the study of some specific problems, such as the difficulty of healing circular wounds. *Physics* 6 covers only one aspect of the study of motion, albeit an important one. Similarly, it does not imply the reduction of the essences of physical things to mathematics, since it deals with continuity only as a property of movement, not its essence. In this, it hardly differs from astronomy. That science, too, subordinates physics to mathematics, but only by considering the heavenly bodies' movements *qua* circular, without concerning itself with their essential nature.

Aristotle discusses the distinction between physics and mathematics in *Physics* 2, $193^{b}22$–$194^{a}12$. There, he is contrasting physics with mathematics in general, yet he provides for subordinate sciences relating them. In fact, the reason he gives for having to draw the distinction is that physical objects do have certain mathematical properties ($193^{b}22$–25). Such is the case in *Physics* 3. $207^{b}21$–27 (quoted above, pp. 287–8), where change derives from extension the properties of quantitative measurability and continuity. Pure mathematics differs from physics because it abstracts mathematical properties totally from the subjects in which they inhere, rather than studying them *qua* inhering in physical subjects ($193^{b}31$–33). But pure physics cannot abstract the properties it studies from their subjects at all ($193^{b}35$–$194^{a}7$). It must be the subordinate sciences, then, that do study abstract mathematical properties *qua* inherent in certain physical subjects:

> δηλοῖ δὲ καὶ τὰ φυσικώτερα τῶν μαθημάτων, οἷον ὀπτικὴ καὶ ἁρμονικὴ καὶ ἀστρολογία· ἀνάπαλιν γὰρ τρόπον τιν' ἔχουσιν τῇ γεωμετρίᾳ. ἡ μὲν γὰρ γεωμετρία περὶ γραμμῆς φυσικῆς σκοπεῖ, ἀλλ' οὐχ ᾗ φυσική, ἡ δ' ὀπτικὴ μαθηματικὴν μὲν γραμμήν, ἀλλ' οὐχ ᾗ μαθηματικὴ ἀλλ' ᾗ φυσική. (194^a7–12)

Now, there can hardly be any distinction between the line as studied by geometry and the mathematical line. But if we substitute 'physical line not *qua* physical' for 'mathematical line', this passage implies that optics studies *qua* physical a physical line not *qua* physical. This seems odd, but in a sense it is true. To study the lines at all, one must consider them apart from the other properties of the subject; still, one may then consider these lines as they apply to certain subjects. So, book 6 studies only the continuity of time and motion, apart from their other properties, but it studies continuity *qua* inherent in motion and time.

Final confirmation of the hypothesis that the method followed in *Physics* 6 is that of a subordinate science can be gleaned from a closer look at *Physics* 5. 3.

Aristotle does not always define continuity as he does in this chapter. He uses several definitions of it, including one which simply defines it as infinitely divisible.[1] As Owen notices,[2] the definitions in 5.3 are arranged so that Aristotle can prove continuity's infinite divisibility rather than simply assume it. This is much more effective, especially against Zeno. But perhaps he also chose this arrangement of definitions to stress the relation of continuity with movement. If continuity had been defined in 5. 3 as infinitely divisible, it could not have been linked with movement definitionally.

Owen compares the chapter with *Parmenides* 148e, from which he thinks it was derived. Plato defines continuity there in terms of *touching*, *successive*, and *contiguous*, but not *between*. Aristotle, too, could have begun with *successive*, which he does, in fact, consider the first principle of this conceptual family (227^a17, cf. p. 283 above). Instead, he not only defines *between*, but takes time to explicate the definition. And as we have already noticed, both the definition and its discussion are in terms of motion.

There are also other indications that the chapter was written deliberately in such a way as to prepare for a subordinate science. We have seen that book 6 assumes only so much of the definitions of change and time as is needed to show their correspondence with extension, i.e., to relate them to continuity. Likewise the definitions in 5. 3, and the subjects in which it says the properties defined inhere, are carefully selected to include both physical and mathematical reference to continuity and to exclude other aspects of both physics and mathematics. The examples of subjects in which continuity inheres are conspicuously physical in 227^a17 ('γόμφῳ ἢ κόλλῃ ἢ ἁφῇ ἢ προσφύσει'), but when Aristotle excludes arithmetic's discrete quantities in 227^a27–32, he is careful not to exclude geometry's continuous ones. And the entire chapter avoids any distinction between place (τόπος, usually used in a physical context by Aristotle) and position (θέσις, usually geometrical), despite awkward consequences. The

[1] He implies a criticism of that definition in *Physics* 200^b18–20, but he reverts to it in 232^b24 and offers it also in *On the Heavens* 286^a6.

[2] Owen, op. cit. 95–6.

equivalence of place and position entails that points can 'touch' (227^a29), i.e., they can 'have their extremities in the same first place'. Since 'first place' for Aristotle is the limit of the containing body, this means not only that points have extremities, but that they are contained by three-dimensional bodies.

The restricted scope of the chapter's definitions was noticed by Simplicius (*ad loc.*). He supposed that 5. 3 gives only definitions which are relevant for physics. But his own example shows that the restriction of relevancy is narrower than that. He compares the chapter's definition of *together* with that in the *Categories*. 5. 3 defines it in place only, while the *Categories* defines it in time and nature also. But time and nature are just as relevant to physics as place is. It is with regard to continuity that *together in place* is paramount.

Similarly, Aristotle restricts the definition of *between* in 5. 3 by defining it in terms of movement continuous in regard to the distance traversed and specifically not in regard to the time (226^b27–31, following Ross's interpretation of 'τοῦ πράγματος'). But elsewhere he determines the continuity of movement precisely by the time (*Metaphysics* 1016^a5–6).[1] The reason for his insistence in 5. 3 on deriving movement's continuity from the distance traversed is that here he must observe the order of priority among extension, movement, and time set up in 207^b21–27 (quoted above, pp. 287–8).

Aristotle's method in *Physics* 5. 3 and 6, then, corresponds to that of a subordinate demonstrative science, providing a specimen of systematic demonstration in the treatises. That is what this article set out to prove. But some further conclusions are implied by what has been revealed in the process.

Aristotle rarely, if ever again, used the method as extensively as here. It is not insignificant that he used it at length only in this part of the *Physics*, where he was concerned with a quasi-mathematical issue and followed the methodology of subordinate sciences. This confirms the belief that he associated the demonstrative method particularly with mathematics.

In *Physics* 1–4, which are later, he is searching to establish basic definitions, not deducing from them. His early passion for systematic deduction, evidenced by *Physics* 6 as well as by the *Analytics* itself (also an early work), has waned.

Perhaps some of the reasons for this will emerge if we consider some conclusions which our analysis suggests about the original version of books 5–6. We have noticed that the assumptions that change is 'from something into something' and 'in time' function in the role of definitions of change and time in these books. We have also noticed that the first of these assumptions is derived from a verbal definition of μεταβολή in book 5, that the second is equated with the definition of time as the measure of change in book 4, and that no other definitions of time or change are stated in books 5–6. The reference to the definition of κίνησις in 224^b10 is a cross-reference which cannot have been in the original version of these early books. And the explanation of 'in time' in book 4, too, must have been written when the different versions of the *Physics* were incorporated. Moreover, the early books show no knowledge of the distinction between potency and act, which is a central theme of the later physics,

[1] The *Metaphysics* passage offers interesting support for my hypothesis. There, *continuous* is 'that whose movement is *per se* one and cannot be otherwise', while 'one' is explained as 'indivisible in time'. That definition of *continuous* is not given in *Physics* 5. 3 because, although it would have surpassed the latter's arrangement by defining *continuous directly* in terms of movement, it would have implied indivisibility rather than infinite divisibility!

and Aristotle can only use that distinction to unravel the solution to Zeno's puzzles in book 8.

Against the charge that a subordinate science treating change and time only as continua would constitute a reduction of physical to mathematical reality, I argued that books 5–6 are but a part of the *Physics*. The original version of these books, however, if taken alone as a complete study of physics, might constitute such a reduction. Aristotle's enthusiasm for demonstrative science as a philosophical method, then, lasted only as long as he was investigating that aspect of the world of change which could be explained mathematically.

Such chronological dissection, however, must not lead us to interpret *Physics* 6 apart from its context. In the final version of the *Physics*, Aristotle left no cut-and-dried deductive system. The systematic deductions of book 6 have their foundations in book 5, are occasionally interrupted in book 6 itself, and continue sporadically in books 7 and 8.[1] In the *Physics* as a whole, Aristotle considers his methods complementary, not mutually exclusive, and combines them on an *ad hoc* basis.

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[1] W. Kullmann, 'Zur wissenschaftlichen Methode des Aristoteles', in Flascher and Gaiser, *Synusia* (Neske, 1965), 247–74, finds occasional demonstrations, which prove the inherence of certain properties in certain subjects by making deductions from previously proved propositions, in the treatise *On the Heavens*. These, too, seem to be largely a continuation of the system deduced in *Physics* 6.